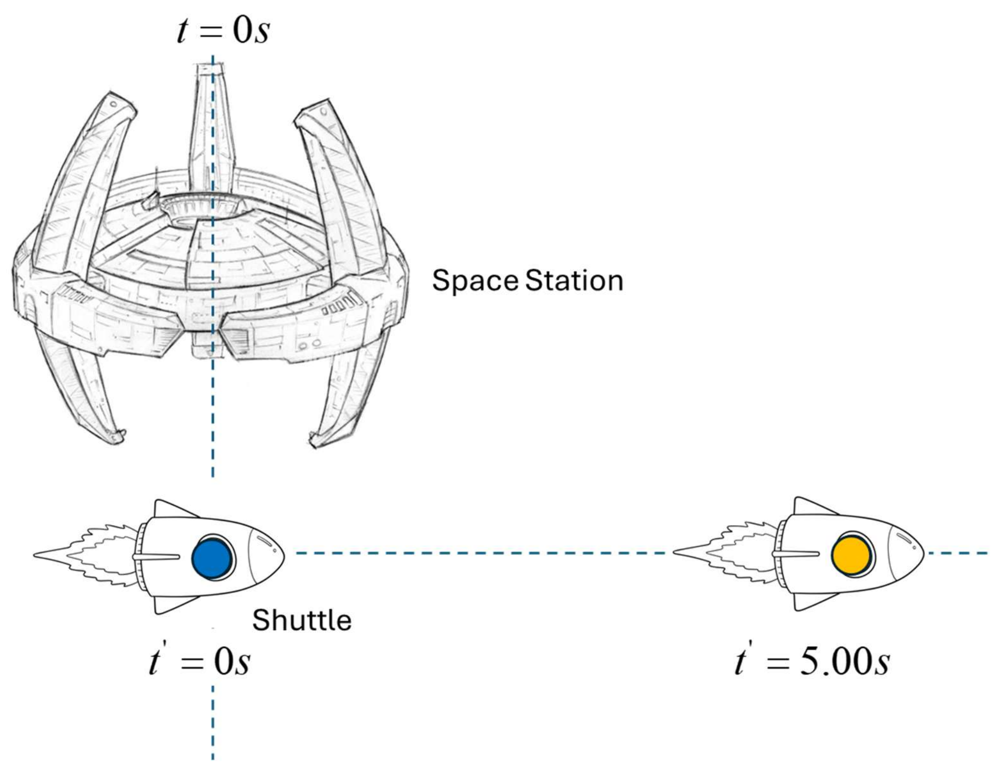
Chapter 37: Relativity

Group Members:

1. A space shuttle zips past a space station at a constant speed of $0.800c$ relative to the space station. When the shuttle passes directly below the space station, both the shuttle pilot and the commander on the station start their own timers at zero and the shuttle pilot turns on and off a blue light inside the shuttle. When the shuttle's timer read $5.00s$, the shuttle pilot turns on and off another yellow light inside the shuttle. Without loss of generality, we will assign the space station as the unprimed "stationary" frame and the shuttle as the primed "moving" frame.



a. Calling the event when the pilot turns on and off the blue light as Event 1. Since Event 1 marks the starting point for both observers and we are free to assign this event to have $x = x' = 0m$ in their respective reference frames.

So, we can set $(x_1, t_1) = (0.00m, 0.00s)$ and $(x_1', t_1') = (0.00m, 0.00s)$.

Now calling the event when the shuttle pilot turning on the shuttle yellow light as Event 2, use the Lorentz coordinate transformation to calculate the position x and time t of Event 2 as measured by the commander on the space station.

The relative speed between the two frames is $u = 0.80c$, so

$$\gamma = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}} = \frac{1}{\sqrt{1-0.80^2}} = \frac{1}{\sqrt{0.36}} = \frac{1}{0.6} = \frac{5}{3}$$

Since the light is at rest on the shuttle, the assignment of the value for x' for Event 2 in turning on and off the yellow light will still be $x' = 0.00m$. Then, the space-time coordinate of Event 2 in the S' frame is,

$(x_2', t_2') = (0.00m, 5.00s)$

Now, using the Lorentz coordinate transform for qualities from $S' \to S$, we can calculate (x_2, t_2) in unprimed S frame,

$$x_2 = \gamma(x_2' + ut_2') = \frac{5}{3}(0 + (0.8c)5.00s) = \frac{5}{3}(4s)(3\times10^8\ m/s) = 2.00\times10^9\ m$$

$$t_2 = \gamma\left(t_2' + \frac{u}{c^2}x_2'\right) = \frac{5}{3}(5.00s + 0) = 8.33s$$

b. Calculate the duration between these two events as measured by the shuttle pilot in S'.

The duration measured by the shuttle pilot is just

$\Delta t' = t_2' - t_1' = 5.00s - 0.00s = 5.00s$

c. Calculate the duration between these two events as measured by the station commander in S.

The duration measured by the station commander is just

$\Delta t = t_2 - t_1 = 8.33s - 0.00s = 8.33s$

d. Which observer, shuttle pilot or station commander, is stationary with respect to both events and measure the proper duration between these two events?

The light being turns on and off is at rest in the shuttle and the pilot is stationary (not moving) with respect to the shuttle so the pilot will measure the proper duration between the two events: Event 1 and 2 and $\Delta t'$ measured by the pilot in S' is the **proper** time interval Δt_p, i.e., $\Delta t_p = \Delta t'$ for these two events.

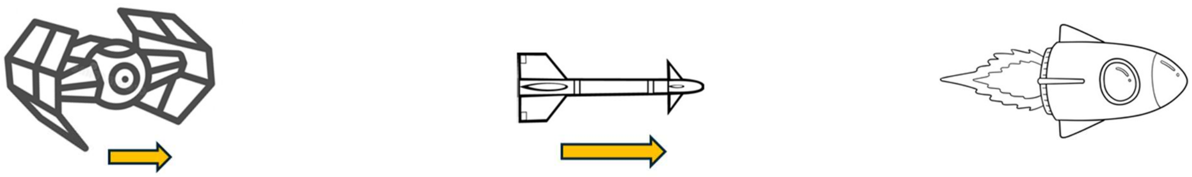
e. As we learn in relativity, observers in different inertial reference frames will measure time durations differently. Use the time dilation formation to calculate the duration between Event 1 and 2 according to the station commander.

Since the station commander is not stationary with the two events, the time interval Δt as measured by the commander is not proper and it will be dilated, longer than 5.00*s*, as measured by the pilot.

$$\Delta t = \gamma \Delta t_p = \gamma \Delta t' = \frac{5}{3}\left(5.00s\right) = 8.33s \; .$$

This has the same value as calculated in part d using Lorentz coordinate transformation.

2. An enemy spaceship is in hot pursuit of your starfighter with a speed measured by you to be 0.500c. The enemy spaceship fires a missile toward you at a speed of 0.850c relative to the enemy ship.



$u = 0.5c$ Enemy Spaceship (S')

$v_x' = 0.85c$ Missile

Starfighter (S)

   a. What is the speed of the missile relative to you in the Starfighter? (Express your answer in terms of the speed of light.)

      First, we define our reference frames. Call the reference frame on the starfighter as S and the reference frame for the enemy spaceship as S'. Now, we can appropriately label the various quantities as measured by the two observers in their respective frames.

      The relative speed between the two frames is $u = 0.500c$

      We are given that the missile speed is measured wrt to the enemy ship (S' frame) and toward you. So, we have,
      $v' = 0.850c$

      Now, we will use the Lorentz Velocity Transformation to calculate the unprime velocity as measured in your S frame,

$$v = \frac{v'+u}{1+\frac{u}{c^2}v'} = \frac{0.850c+0.500c}{1+\frac{0.500c}{c^2}(0.85c)} = \frac{1.35c}{1+1.35} = 0.947c$$

b. If the enemy spaceship as measured by you to be at a distance of $8.00 \times 10^9 m$ from you as the missile was fired, how much time as measure in your frame, will it take the missile to reach you?

You can simply use $\Delta t = \frac{L}{v}$ to calculate the time Δt which it takes for the missile to reach you. [Using a physical law as in this case, all qualities must be measured in the SAME frame.]

We have $L = 8.00 \times 10^9 m$ measure by you in S and $v = 0.947c$ is also the velocity of the missile measured by you in S. [Don't use 0.85c since that is the velocity of the missile measured by your enemy in a different frame.]

$$\Delta t = \frac{L}{v} = \frac{8.00 \times 10^9 m}{(0.947)\left(3.00 \times 10^8 \, m/s\right)} = 28.1s$$

c. To retaliate, you send another missile toward the enemy spaceship with a speed of 0.85c as measured by you. What is the velocity of the missile as measured by the enemy spaceship? [Play attention to signs and directions.]

In terms of directions, your missile will be traveling away from you in the opposite direction as the enemy spaceship (toward you). So, wrt your S frame, $v = -0.850c$

Now, we use the *inverse* relativistic velocity transform to calculate v' from v,

$$v' = \frac{v - u}{1 - \frac{u}{c^2} v} = \frac{-0.850c - 0.500c}{1 - \frac{0.500c}{c^2}(-0.85c)} = \frac{-1.35c}{1 + 1.35} = -0.947c$$

3. Calculate the speed of a particle whose Relativistic Kinetic Energy is exactly half of its Total Relativistic Energy. (Express your answer in terms of c.)

The Relativistic Kinetic Energy is given by the formula,

$KE = (\gamma - 1)mc^2$

And the Total Relativistic Energy is given by the formula

$E = \gamma mc^2$

Now, setting the condition, $KE = E/2$, we have

$$(\gamma - 1)mc^2 = \frac{\gamma mc^2}{2}$$

Canceling mc^2 on both sides and solving for γ, we have,

$$\gamma - 1 = \frac{\gamma}{2}$$

So, $\gamma = 2$.

Now, from the definition of γ, we can express v in terms of γ,

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Squaring and inverting both sides, we have,

$$1 - \frac{v^2}{c^2} = \frac{1}{\gamma^2}$$

Now, solve for v/c, we have,

$$\frac{v}{c} = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \frac{1}{2^2}} = \sqrt{\frac{3}{4}} = 0.866$$

or, $v = 0.866c$